

COMPARISON OF THE ACOUSTICAL PERFORMANCE OF MOSQUE GEOMETRY USING COMPUTER MODEL STUDIES

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ABSTRACT

Speech intelligibility is a major concern in mosque acoustical design. Diverse mosque geometric configurations exist varying from the classical rectangle to the octagon-shaped plan. This study investigates the acoustical performance of commonly built forms of mosques utilizing room-acoustics computer models. Simulation of sound fields of five simple forms is conducted for different religious activities and level of occupancy. The purpose is to identify the impact of the mosque geometry on its acoustics, particularly on the spatial distribution patterns of speech intelligibility in the absence of sound reinforcement systems. Speech intelligibility contours were quantified and compared to characterize acoustic merits, dissimilarities and overall performance. Insignificant differences were found. Of all the shapes, (standard designs) the octagonal mosque possesses the fewest merits. The investigation is expected to help architects to understand better the effect of early architectural design decisions with respect to form on mosque acoustics.

INTRODUCTION

Acoustic modeling and simulation are beneficial and effective computer-based tools. Room-acoustics programs have typically been used for the prediction and assessment of room acoustic indicators in the early design stage of various spaces. For example, the prediction of echograms and impulse responses within enclosures utilizing "Epidaure" software was described (Maercke and Martin, 1993). The "Odeon" acoustics simulation package (ODEON, [www](http://www.odeon.dk)), a combined beam tracing and radiant computer model of room acoustics, was developed (Naylor, 1993). Using such computer modeling and simulation, problems such as echoes can be identified and the overall acoustic performance of spaces intended for particular functions can be assessed before actual construction begins (Claus, 2001). The impact of architectural design decisions can be readily visualized and even listened to, developing an appreciation of the results via a cause and effect type of analysis. In addition, computations of introducing

sound reinforcement systems using multiple electro-acoustic sources can be investigated at the early design stages. As well as the ODEON program, CATT, CARA, RAYNOISE, AURORA, RAMSETE, and NEMPEE (web sites) are a few examples of similar acoustics simulation software packages with a variety of features and merits.

Nowadays, as a result of continuous research and development, room-acoustical computer models have significantly improved becoming reliable and efficient design tools for acoustical investigations. Rindel (2000) well described the various methods of simulating sound in rooms. Earlier, Abdou (1999) employed such tools for predicting and assessing the acoustical performance of mosques. Recently, computer simulations of the acoustics of mosques and Byzantine churches were also conducted (Wetiz et al., 2001). Moreover, acoustic computer simulations were carried out for Hagia Sophia, which is characterized as one of the largest buildings of worship in the world. The room acoustic differences in the three historical periods when the building was used as a church, a mosque and, currently, as a museum were investigated by creating three different computer models. Each included the particular function and relevant furnishings. Wetiz et al. (2002) applied the acoustic computer simulations on some of the old churches and mosques in Istanbul for the purpose of comparing between in-situ recordings and auralization obtained from simulations utilizing the ODEON program.

GEOMETRY OF MOSQUES

Mosques possess basic common design features as spaces for worship (King, 1986, and Sergeld, 1996). The mosque is typically a simple rectangular, walled enclosure with a roofed prayer-hall. The long side of the rectangle is always oriented towards the holy mosque in *Makka* city in Saudi Arabia. This wall called the "*Qibla*" wall is always emphasized by a central niche (called "*Mihrab*"). To its right an elevated floor (called "*Minbar*") is used by the preacher (i.e., the "*Imam*") to deliver the religious "Friday" speech (i.e., "*Khutba*") preceding the prayers. *Figure 1(a)* illustrates the basic elements of the mosque space design.

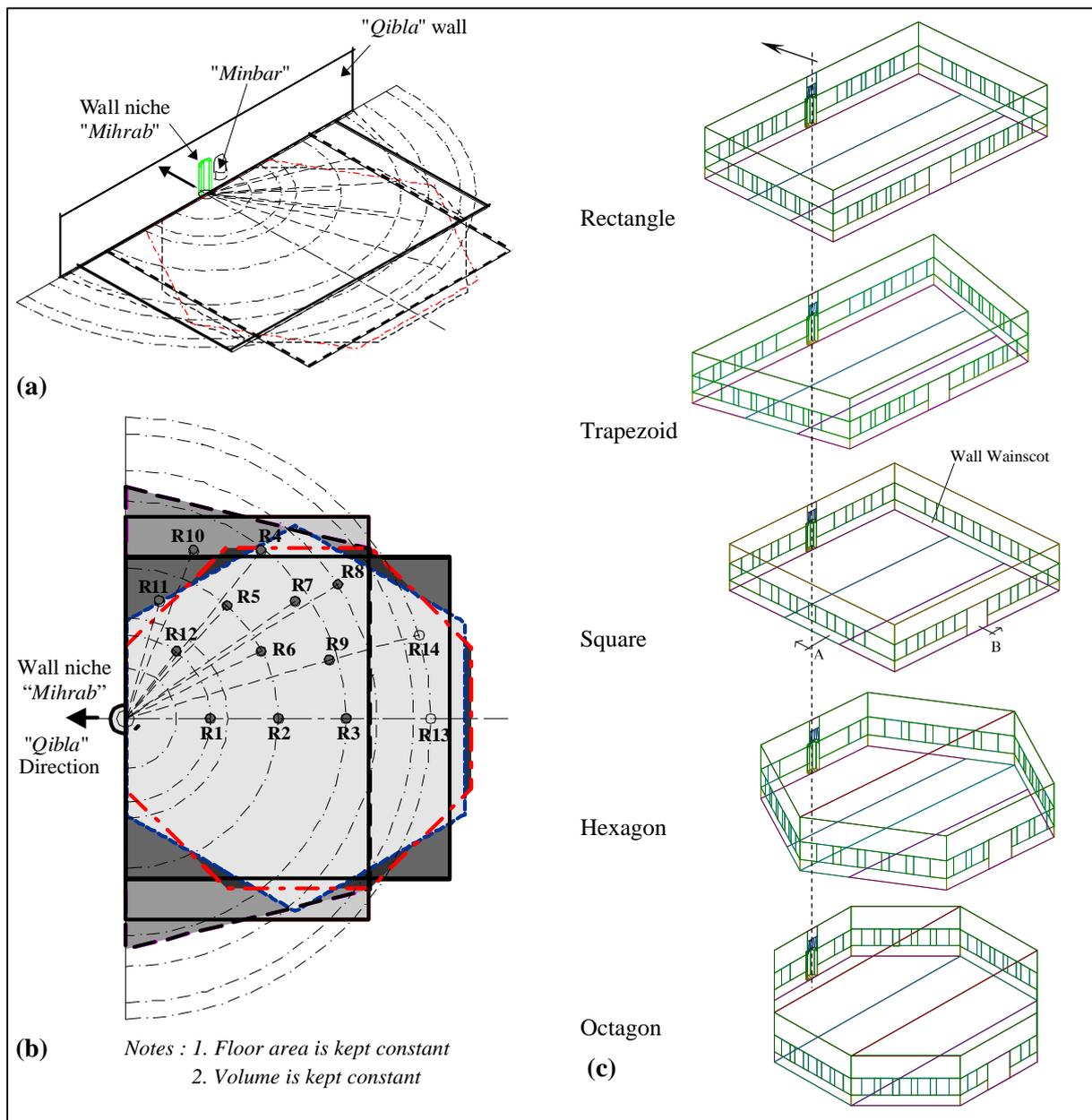


Figure 1 (a) The main design elements and features of the mosque prayer hall, (b) The investigated mosque geometry overlaid, and (c) 3D illustrations of the modeled mosque forms.

Interior materials and finishes of mosques vary from one country to another. However, mosque walls are commonly finished with painted plaster. Wall wainscots are sometimes covered with marble tiles or wooden boards or panels tongued and grooved to compose a vertical pattern. The floor area is always carpeted. Plastered and painted concrete ceilings with simple to elaborate decorations and /or inscriptions are commonly used. Depending on the climatic conditions, the mosque may be equipped with an air-conditioning system, in concert with some ceiling fans. Electro-acoustic sound reinforcement systems have also been installed in mosques of all sizes to improve the hearing conditions in the space, particularly when air-conditioning systems are

ACOUSTICS OF MOSQUES

The acoustics of a room are commonly judged by their reverberance evaluated from the sound level decay curves. The first, conventional Reverberation Time (RT), is defined as the time it takes for sound to decay by 60 dB after the sound source has stopped. It is usually determined by extrapolating the slope of a straight line fitted to the first part of reverberant decay curves as a function of frequency between -5 and -25 (RT₂₀), or -35dB (RT₃₀). The second indicator is the Early Decay Time (EDT), which is found to be a subjectively more relevant indicator than RT and is defined as the sound decay slope of a straight line fitted to the decay observed during the first -10 dB. EDT values are more influenced by the

details of early reflections. Both measures indicate reverberance as a function of frequency, which in turn appears to be responsible for the sensation of being in a room as well as providing a sensation of distance from the sound source. Optimal RT values depend on the desired function of the hall, the hall volume and interior surface finishes. For optimum listening conditions for speech intelligibility RT values must be in the range of about 0.5 to 1.0 second at mid-frequencies (i.e., average of RT at 500 and 1000 Hz one-octave bands i.e. RT_m). Preferred ranges of RT values at mid-frequencies for a variety of activities are well established. For example an RT value of less than 1.0 sec is desired for an intimate drama theatre, lecturing or for speech activities while 1.8 sec is acceptable if the space is to be used as a multi-purpose hall (Templeton et al., 1997).

The effect of the enclosure acoustics on speech intelligibility (SI) is found to be better related to the beneficial sound energy of the direct sound and reflections arriving within the 50 ms after the direct sound. Late arriving reflections, indicated by a long reverberation time, reduce intelligibility. The subjective balance between sound clarity, definition and reverberance can be judged by the arriving early-to-late sound ratio indicators such as Clarity (C_{50}), defined as the ratio of the early arriving sound energy in the first 50 ms after the direct sound to the late sound energy arriving afterwards. Clarity is increasingly considered more indicative of the impact of the room on SI. For clarity and blend balance expressed by the foregoing indicators, a low value indicates poor definition, referred to subjectively as “muddy” sound. While a high value indicates that it is possible to discriminate the sound details, the sound may also be subjectively “very dry” as if it is produced in a room with too much absorption.

Unlike the acoustical characteristics and requirements of other religions’ spaces, which require the design for both speech and music, speech intelligibility is the only major concern of mosque acoustics. The intelligibility of speech in a mosque is essential to the performing of prayers and related religious activities. All activities in the mosque are dependent on speech audibility and intelligibility. These two factors are, thus, critical to the evaluation of sound quality in a mosque. Knowledge pertinent to mosque acoustics in this regard, compared to that of other religious enclosures where speech intelligibility is also important, has received very little attention. The intelligibility of speech in rooms is related to both the speech sound level and to the ambient noise i.e. expressed by speech signal-to-noise ratio and to the acoustical characteristics of the space indicated by reverberation time. The less the room reverberance and the higher the level of the speech sound relative to the ambient noise, the greater the intelligibility of the speech. Many indicators can be used in order to

measure, assess and/or predict speech intelligibility (Johan, 1997). Examples of objective-based measures are RT, Definition (D_{50}), Clarity (C_{50}), Useful-to-Detrimental Sound Ratios (e.g. SNR_{95} , U_{50} , Bradley, 1986), Speech Transmission Index (STI), Rapid Speech Transmission Index (RASTI) and Articulation Loss of Consonants ($\%AL_{cons}$) (Putez, 1971). In this study the STI indicator used for assessment will be reported.

METHODOLOGY OF MODELING AND SOUND SIMULATION

The preferred shape of the mosque is usually the architect’s decision, influenced by the Islamic values, teachings and the way the prayer is performed both individually and in group as religiously prescribed. Group prayer must be performed with individuals standing, behind the *Imam*, in straight rows around 1.2 m apart and parallel to the *Qibla* wall. Consequently, it is desired that the mosque shape be bounded by straight parallel walls, one of which includes the *Qibla* niche. Religious preference is higher for those praying in the first rows compared to late arriving individuals. Hence first rows are preferred to be longer or at least equal to the subsequent remaining ones. Rectangular and trapezoidal plans with the long side perpendicular to the direction of the *Qibla* well satisfy these preferences. The square shape is also acceptable. Nonetheless, the hexagon and octagon shapes violate this preference, yet hexagonal and octagonal mosques were designed and built in many parts of the Muslim world. These five shapes were acoustically modeled for comparison. *Figure 1(b,c)* shows the architectural forms and the main features of mosque geometric configurations that were investigated. These prototypes can be considered to be medium-size, community mosques with a mean volume of approximately 1659.0 m³. Since a comparison of the acoustical impact of alternative mosque geometry is the subject of this study, the mosque geometric parameters such as volume, floor area, walls and windows areas and ratios were kept constant for a valid comparison of the impact of the mosque geometry. *Table 1* shows the geometric information of the different mosque shape. Since the room volume influences RT, the volume of the modeled mosque was kept constant with a standard deviation of ± 2.0 m³. The capacity of the mosque is determined by the floor area divided by the area required for a worshipper to perform the prayer, i.e. about 0.80 x 1.2 m². As can be observed from *Table 1*, the mean floor area is 350.0 m² with ± 10.0 standard deviation. This average area can accommodate approximately 350 worshippers when fully occupied. Windows are modeled to be around 15% of the mosque floor area and 15% of the total surface area of the mosque walls. They were uniformly distributed in the walls.

Table 1
Geometric information of mosque shapes (note values are rounded to nearest integer)

Shape	Dimensions (W, L, H, m)	Floor Area m ²	Volume m ³	Wall Surfaces m ²	Windows		
					Area, m ²	To Wall Area %	To Floor Area %
Rectangular	14.40 x 24.00 x 4.80	346	1659	387	56	14	16
Trapezoidal	14.40 x 27.00 x 4.80	346	1659	373	57	15	16
Square	19.20 x 19.20 x 4.50	369	1659	369	52	14	14
Hexagon	Side = 11.54, H = 4.80	346	1662	332	50	15	14
Octagon	Side = 8.45, H = 4.80	345	1656	325	52	16	15
Mean		350.0	1659.0	357.0	53.0	15.0 %	15.0 %
Standard Deviation, STD		±10.0	±2.0	±27	±3	±1	±1

Table 2
Material assignment for interior surfaces of all mosque configurations

Surface	Assigned Material	Diffusion Coefficient
Ceiling	Lime, cement plaster	0.25, with beams
Floors	9 mm tufted pile carpet on felt underlay	0.15
Walls	Concrete blocks with plaster, painted	0.10
Wall Base (height 1.0 m)	Cladding of marble tiles (see <i>Figure 1</i>)	0.10
<i>Qibla</i> wall niche (<i>Mihrab</i>)	Ceramic tiles with smooth surface	0.10
Windows	Single pane of glass, 3 mm	0.10
Door	Solid wooden doors	0.10
Congregation (worshippers)	Congregation performing prayers standing in rows 1.20m apart.	0.70

The mean values and the variations of the geometric parameters among the five mosques expressed by the standard deviation are shown in the *Table 1*. Similarly the interior surface finishes of the mosque were selected to mimic typically modest finishes commonly applicable in mosques as constructed in practice. The same the interior surface materials were also assigned to all mosque configurations. *Table 2* indicates the selected finish materials to the mosque wall surfaces and interior architectural features along with their assigned diffusion coefficients. The diffusion coefficient of the ceiling is assumed to be 0.25 to account for the existence of intersecting beams carrying the roof slab.

Two different worship scenarios were examined by the simulation program. In the first, Scenario A, the following conditions are assumed: The congregation (worshippers) is performing the prayer behind the *Imam* who is reciting in a standing position facing the *Qibla* niche using his raised voice. It is natural that persons delivering speech without the aid of Electro-acoustic sound system tend to raise their voice. The background noise in the mosque is assumed to reach a Noise Criterion (NC) rating of NC25 as recommended in enclosures designed for speech. The worshippers are assumed to be also standing listening to the *Imam* as is usually the case during performing the “*Daily*” prayers. Their ear height is taken to be 1.65 m from the floor.

In the second, Scenario B, the *Imam* is assumed to be delivering the *Friday* speech in a raised voice,

without the aid of sound reinforcement system, from the *Minbar* which is elevated about 1.25 meter from the mosque floor. His mouth height is around 2.80 m from the floor. The worshippers are assumed to be seated on the floor listening to the speech as is usually the case during *Friday* prayer. Their ear height is taken to be 0.80 m from the floor.

As mentioned earlier, knowing the volume of the modeled mosque, it is possible to estimate the required optimum RT_m for good SI. For the given mosque volume (i.e., 1659.0 m³) the optimum RT_m value for speech purposes should be in the range from 0.6 to 1.2 sec (Duncan et al., 1997). For comparing the different geometry, 9 to 12 worshipper locations, representing different zones of the mosque, were investigated to study the positive impact or otherwise of the mosque form on SI. These locations are indicated in *Figure 1(b)* and denoted R1 to R12. GIRD calculations were also conducted. Sound quality indicators such as EDT, A-weighted sound pressure level (SPL(A)), C_{50} and STI were used for comparisons. However only STI are reported in this study. To make sure that the five mosques have similar reverberance, the RT values based on the classical “Sabine” RT_{Sabine} , and “Eyring”, RT_{Eyring} formulae were considered in both the unoccupied and fully occupied state of the mosque. *Figure 2* depicts RT vs. octave-band frequencies for the five mosques determined according to the Eyring Formula when the mosque is assumed empty, 1/3 occupied and fully occupied. As can be seen from the figure, as intended, the five mosques possess similar RT spectra

in the three levels of occupancy, with some minor variations at low frequencies when the mosque is 1/3 occupied. Neither the “Sabine” nor the “Eyring” formulae takes into account the spatial distribution of the sound absorbing-materials but they both give a sense of the absorption magnitude similarities or otherwise.

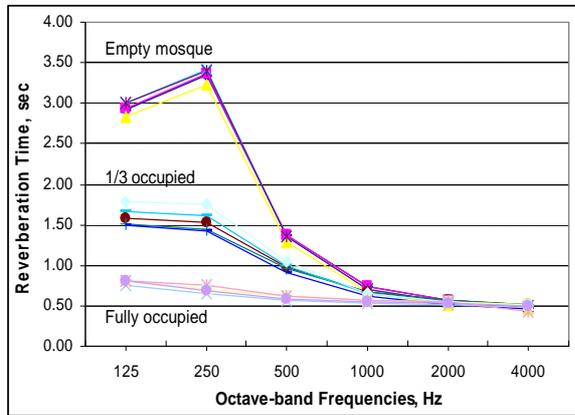


Figure 2 RT vs. octave-band frequencies for the mosque geometry determined according to “Eyring” Formula when the mosque is assumed empty, 1/3 occupied and fully occupied.

COMPARISON OF SOUND FIELDS AND ACOUSTICAL MERITS

The results of the simulations in terms of spatial distribution patterns (SDP) of STI are depicted in Figure 3. Values of STI are calculated on a grid of 1.0 x 1.0 meter for the occupied mosques when the worshippers are performing “Daily” group prayers while the Imam is facing the Qibla niche, are shown in part (a) of the figure. STI ratings are indicated on the value colored scale bar in each case. The zones in the mosque which exhibit STI values less than 0.5 are the dark-colored grids. For clarity, they are highlighted with bold bounds. As can be seen, the zones of “Fair” STI rating are located in the center of the rear half of the floor area with some other “Fair” zones near the middle of side walls and the far front corners. The SDP of STI in the trapezoidal mosque show signs of the same patterns as in the case of the rectangular one with side “Fair” zones expanded to the Qibla wall. The “Fair” zones however, cover almost two thirds of the floor area in the square mosque, with the STI rating in the rear third almost approaching the “Poor” rating. The STI distribution pattern in the front one third of the floor is uniformly distributed.

Examining the Hexagon mosque reveals that the “Fair” zones are concentrated in the middle and rear parts of the mosque.

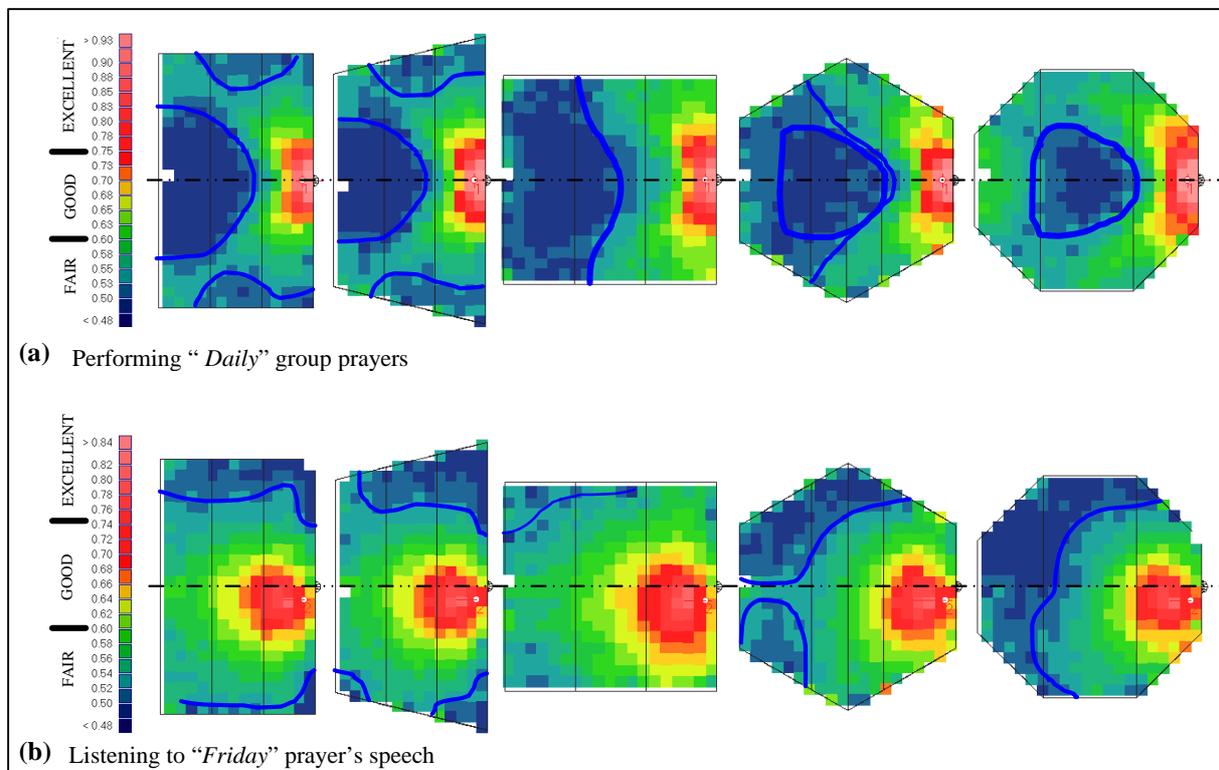


Figure 3 Spatial distribution patterns of STI assuming a background noise level and spectrum of NC-25. Values are simulated for fully occupied mosques when the worshippers are (a) performing daily group prayers, when the Imam is facing the Qibla niche, and (b) sitting on the floor carpet listening to the Imam

Zones with “Good” SI ratings and above are mainly located near the *Qibla* wall and parallel to the two front sidewalls. On the other hand, the octagon-shaped mosque reveals “Good” STI all around the eight sidewalls with the zone confined to the middle with “Fair-to-Poor” ratings.

Part (b) of *Figure 3* shows the distribution patterns of STI of those sitting on the floor carpet listening to the *Imam* (standing on the *Miharb*) delivering the *Friday* prayer speech. The SDP of STI in this worship mode is quite different and, since the sound source is high and the listeners’ ears are near the floor, much of the direct sound energy is reaching almost all worshippers in the mosque. There are, therefore, fewer “Fair” SI zones. One may also notice the extension of the “Excellent-Good” SI ratings near the *Imam* as he is facing the worshippers compared to similar zones shown in part (a) of the figure in case the *Imam* is not facing the worshippers. Additionally, the *Imam* (source) location is not located on the main axis of the mosque as in the first Scenario. The square mosque shows minor areas of “Fair-Poor” SI. The SDP of SI for part (a) of *Figure 2* was then transformed to STI contours as shown in *Figure 4*. Only zones with “Excellent-Good” ratings are bounded to facilitate the examination of those floor areas compared to others with lower SI ratings. Such areas are larger in the square and octagon mosques compared to the other geometry. From the statistical viewpoint, *Figure 5* depicts the percentage listeners (%) in each of the five STI rating categories i.e. “Excellent”, “Good”, “Fair”, “Poor”, and “Bad”. *Figure 5(a)* illustrates the results of the simulation of Scenario A. In this case almost 25% of the listeners have “Excellent-Good” SI ratings, 80% of the worshippers have “Fair” SI, while 5% of the listeners in the octagon mosque encounter “Poor” SI. The results of simulating Scenario B indicated that the “Excellent-Good” SI locations are above 30% for the mosque floor area in the square, hexagon and octagon mosques representing the highest among the five mosque configurations.

It is important to examine the behavior of STI on the axes perpendicular and parallel to the *Qibla* wall. *Figure 6(a)* illustrates STI values vs. distance from the sound source to the rear rows of worshippers. As can be seen up to almost 3.5 meters behind the *Imam* STI decreases linearly from a value of slightly above 0.9 to 0.55. As expected no differences are found in this region where direct sound dominates the sound fields and the subsequently the mosque geometry has little or no impact on STI values. Beyond 3.5 m behind the *Imam*, variations of S/N can be clearly observed despite the fact that these variations lie within the “Fair” rating of SI. The central area of the octagon mosque shows “Poor” SI 7.5 to 10.5 meters from the *Imam*.

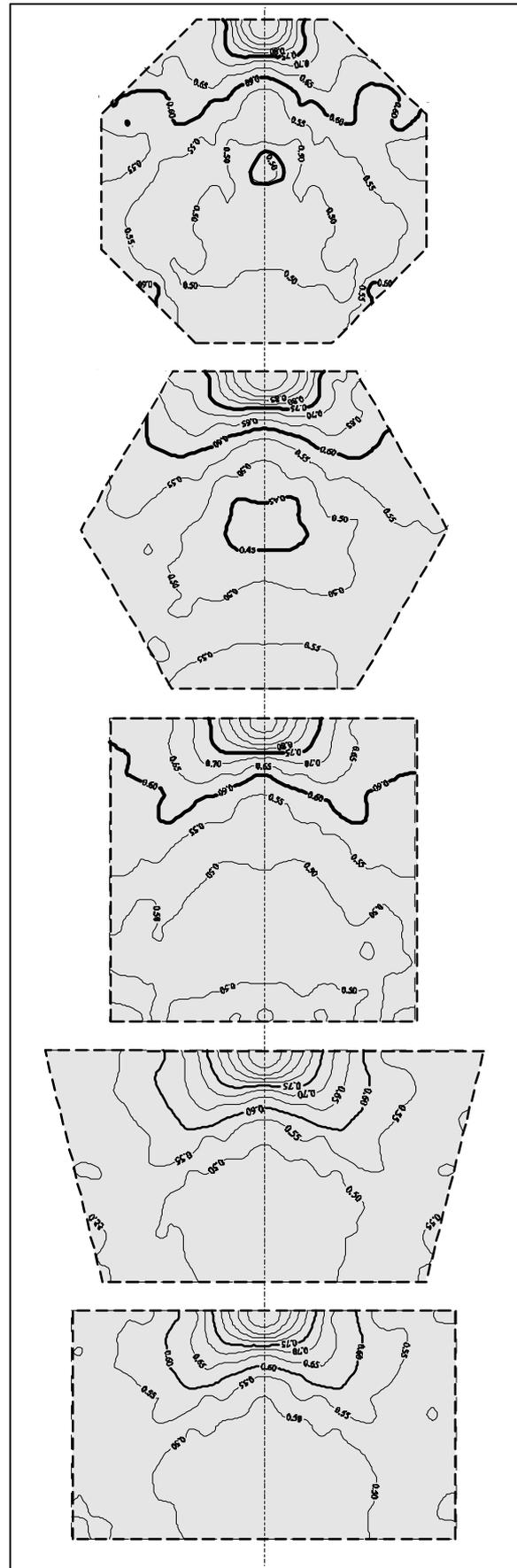


Figure 4 STI contour determined in case of “Daily” group prayers in the five mosque geometries.

STI values start to increase gradually to the rear rows due the effect of reflected sound energy as a result of the side and back walls. Rear rows of worshippers enjoy “Good” STI ratings equal to those found in the first 3-4 rows. Values increase to upper limit of the “Good” rating limit. Rear rows in both the rectangular and trapezoidal floor plans have uniform and constant STI values. Rows in the square, hexagon and octagon mosques exhibit an increase in STI values due to nearby boundaries as beneficial reflections contribute to the received sound energy at listeners’ ears.

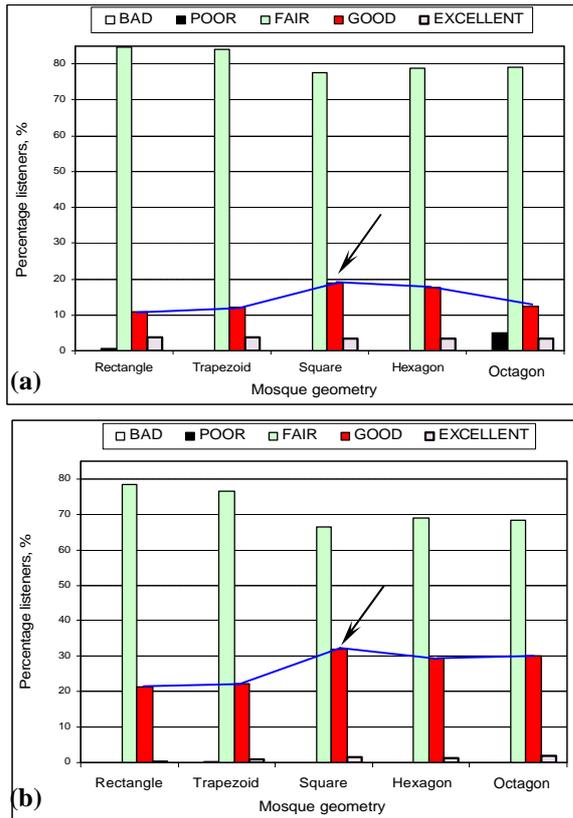


Figure 5 Comparison of STI rating categories in terms of percentage listeners when (a) performing “Daily” group prayers, and (b) sitting on the floor carpet listening to the Imam delivering the “Friday” prayer’s speech.

From examining part (b) of Figure 6, one can observe the behavior of STI in the first row parallel to the *Qibla* wall just behind the *Imam* versus more distant worshippers on both ends of the row. The STI profiles emphasize the symmetry of the investigated geometric configurations. The impact of the Hexagonal sidewalls on the increase of STI is evident. Although the RT spectrum was made constant for all the modeled mosques by controlling the mosque volume and the magnitude of the sound-absorbing materials of the interior surface finishes, when considering the distribution of the absorptive materials in terms of mounting location relative to the

worshippers, simulated RT vary from one worshipper’s location to the other. Figure 7 depicts the global RT spectra obtained from the five mosque models in the case of one-third occupancy, as is usually the case during performing “Daily” prayers. It shows similar RT spectra with variable values. The geometry of the octagon mosque resulted in the highest RT values at all octave-band frequencies particularly at low frequencies (i.e., 125-250 Hz). Global RT values in the hexagon became the second highest. This is also expected in round enclosures or cylindrical forms as reflected sound from boundaries add to the reverberant sound fields. The square geometry resulted in the lowest RT values in the mid-frequencies range (i.e., 500-2000 Hz) where most of the speech sound energy is dominant.

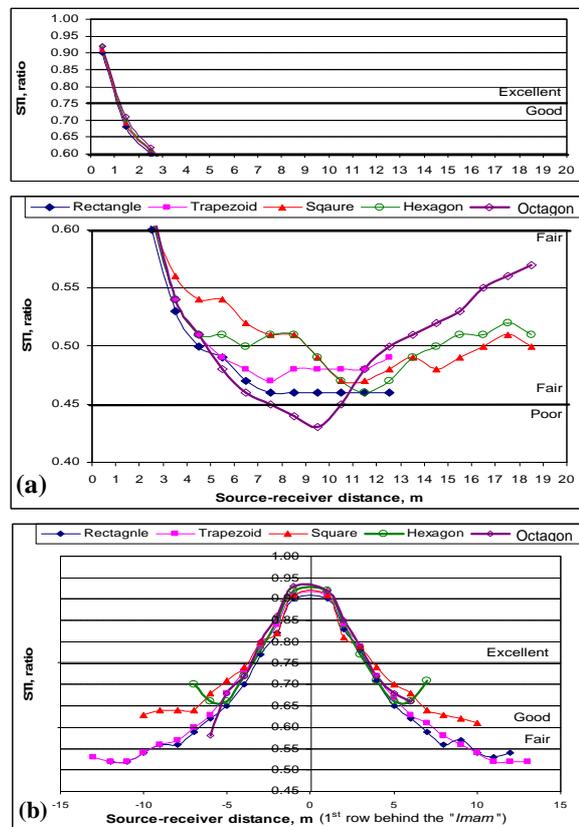


Figure 6 (a) STI ratio and rating versus source-listener distance along the axis (a) perpendicular to the *Qibla* wall, and (b) parallel to the *Qibla* wall along the 1st row of the worshippers.

Knowledge of the magnitude and the most likely spatial distribution of EDT, SPL(A), and STI in the modeled mosques can further help in the process of deciding on remedial measures concerning the installation of sound reinforcement systems. “Poor-Bad” zones with insufficient unamplified sound level can be readily visualized. Consequently the specifications of the power, number, directivity and most importantly the location of loudspeaker required to overcome the resulting acoustical deficiencies can be guided by the information at hand.

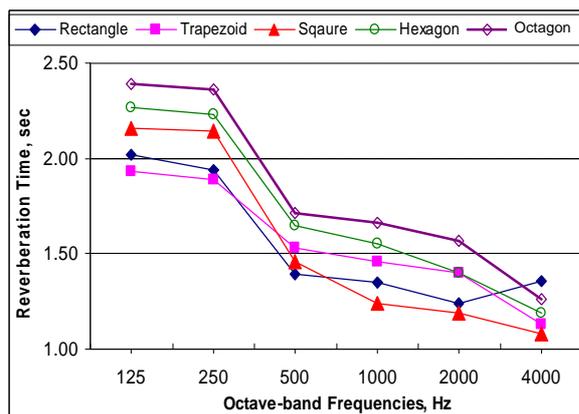


Figure 7 RT spectra in the simulated five mosque geometric configurations

CONCLUSION

Sound fields of five simple mosque geometries were simulated. Reverberation time spectra were made constant by controlling the mosque volume and the magnitude of the sound-absorbing materials used. The impact of the geometry on the ratio of the speech signal-to-ambient noise was then visualized as indicated by the values of STI. Despite the fact that no major differences were found, the square mosque showed the merits of uniform spatial distribution of STI over the front half floor area in the case of worshippers performing “Daily” prayers and almost over the whole floor area when the worshippers were assumed to be listening to “Friday” speech while sitting on the floor carpet. The octagon mosque geometry negatively impacted sound fields in the central zone of the floor area due to the cancellation of sound energy arriving from opposite directions. The investigation carried out in this study is expected to help architects to understand better the effect of early architectural design decisions pertaining to the space and form of the mosque on its acoustics. The spatial distribution of many sound quality indicators can be visualized and assessed. It can also assist the design and installation of sound reinforcement systems in terms of number of loudspeakers required, their directivity, and relevant locations to overcome insufficient sound levels or poor audibility hindering speech intelligibility.

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